Journal of Economic Behavior & Organization 73 (2010) 406–417

Contents lists available at ScienceDirect

Journal of Economic Behavior & Organization

journal homepage: www.elsevier.com/locate/jebo

Price dispersion and increasing returns to scale☆

David M. Levy[a], Michael D. Makowsky[b,*]

[a] Department of Economics, George Mason University, United States
[b] Department of Economics, Towson University, United States

ARTICLE INFO

Article history:
Received 28 April 2008
Received in revised form 7 October 2009
Accepted 9 October 2009
Available online 23 October 2009

JEL classification:
C63
L11
O33
D83

Keywords:
Increasing returns to scale
Price dispersion
Search

ABSTRACT

We present a model in which price dispersion allows the market to remain competitive in the long run amidst increasing returns to scale. The model hinges upon turnover in the productive technology-leading firm, price dispersion resultant of Stigler's logic of rational search, and limited excludability of knowledge. Price dispersion, traditionally viewed as an efficiency loss derivative of imperfect information in the market protects competition from being destroyed by innovation and increasing returns to scale. Bankruptcy occurs in a form similar to the gambler's ruin. The model requires no entry or replacement of failed firms. The number of active firms in a market reaches a stationary state increasing with, and contingent on, search costs.

> "Professor Chamberlin was right in concluding that perfect competition is a poor instrument in analyzing selling costs. His results might have been more informative, however, if he had chosen to drop the assumption that the economy was stationary, rather than the assumption that the economy was competitive." – George J. Stigler (1949)

1. Introduction

Questions of economies of scale suppose a long run in which all inputs can vary. We propose a model in which price dispersion supports and is supported by a competitive, technologically dynamic industry out of which increasing returns to scale (IRS) emerge. Price dispersion, traditionally viewed as an efficiency loss derivative of imperfect information in the market, can be an intertemporal safety net that protects competition from being destroyed by innovation. A market populated by consumers with imperfect information can engender price dispersion sufficient to allow less productive firms to survive long enough to continue participating in an innovative race where technological leadership is in constant flux. This turnover at the top keeps competition robust, even under conditions of increasing returns to scale.

When search costs are zero the relationship among firms with different production costs is simple; only the low cost firm survives. With positive search costs, if the identity of the low cost firm does not change over time, then consumers

☆ The authors thank Sandra Peart, Steven Durlauf, Yew-Kwang Ng, Omar Al-Ubaydli, participants from the Society for Economic Science with Heterogeneous Interacting Agents 2007 meeting, and two anonymous referees for helpful comments. Levy thanks the Pierre F. and Enid Goodrich Foundation and Makowsky thanks the Earhart foundation for its generous funding.

* Corresponding author. Tel.: +1 410 704 2187.
E-mail addresses: davidmlevy@gmail.com (D.M. Levy), mikemakowsky@gmail.com, mmakowsk@gmu.edu (M.D. Makowsky).

0167-2681/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.jebo.2009.10.004

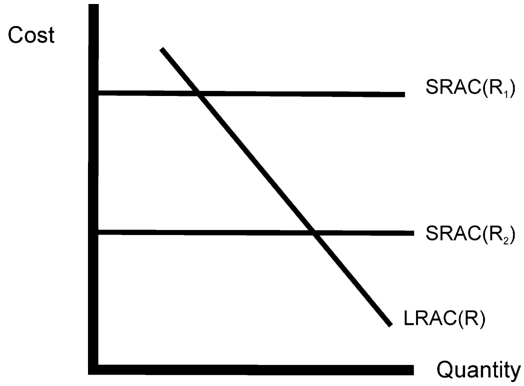

Fig. 1. Knowledge and costs of production.

will eventually find this uniquely low cost firm, again resulting in a single surviving firm. However, if the identity of the low cost firm changes through some form of leapfrog competition, costly search can support a distribution of firms with different production costs. Our paper looks into the complexity which can ensue when producing at the lowest cost is neither necessary nor sufficient for survival, and asks what if survival depends upon a technological competition conducted by fair rules and avoiding the ill fortune of being undiscovered by customers?

Increasing returns are problematic under standard competitive conditions because marginal cost is everywhere less than average cost. Monopolistic competition solutions employ market power mechanisms to shift pricing to average costs. We propose a competitive alternative that offers a bridge between the short run and a competitive long run characterized by IRS. Our model is a variation on Viner's cost curve analysis in which our short run is defined by a fixed stock of knowledge instead of his fixed stock of capital (Viner, 1931).

With this specification of a technological short run, we can then offer a Vinerian interpretation of the early post-World War II studies of knowledge and productivity which employed a production function exhibiting, over labor (L) and capital (K), constant marginal and average costs which were shifted by the stock of knowledge (R) (Terleckyj, 1974). If we suppose a short run in which L and K exhibit constant returns to scale (CRS) with a fixed R, then Fig. 1 describes two short-run average cost curves ($SRAC_1$ and $SRAC_2$), each with a fixed R where $R_2 > R_1$—and the long-run average cost (LRAC) in which R varies over all values.

Firms in our model have incentive to generate knowledge as means both to increase their probability of survival from one short run to the next and to generate short-term rents on technological improvements. Firms with inferior knowledge stocks run the risk of failure. This competitive generation of knowledge creates a declining long-run average cost curve. Without the disruption caused by innovation, price equals marginal cost in the short run, leaving firm size indeterminate and unlimited. The question we address is whether multiple firms can survive a short run in which there is a unique low cost firm. For ease of exposition we focus on survival by taking knowledge generation as exogenous.

Adam Smith saw a world with falling costs derivative of the division of labor and a competitive market where prices oscillated around a "natural" price. The apparent tension between these two concepts in Smith is discussed at length by Warsh (2006). The tension, we argue, results from the employment of Jevons' puzzling Law of One Price (Levy, 2007). We return to Smith's view in which buyers will accept only the lowest price they perceive. In centering growth on innovation, endogenous growth theory has invited questions regarding the impact of innovation on competition and, just as importantly, the reverse. Schumpeterian insights predict that increasing market competition within an industry reduces incentives to innovate (Aghion and Howitt, 1992). More recently Aghion et al. (2001) constructed a model of industry duopoly in which innovation is robust to competition. In the Aghion, Harris et al. model the substitutability of competing goods is a testable parameter; they were able, in effect, to test the impact of competition along a pure monopoly—perfect competition continuum, on innovation and growth. Growth is shown to be monotonically increasing with competition under a variety of conditions,[1] corresponding with recent empirical evidence (Blundell et al., 1995, 1999).

We seek to construct a partial equilibrium model of m firms in a single industry able to draw resources from the rest of the economy in which competition is robust to IRS. We will test our model under conditions of both long-run IRS emergent from innovation as well as short-run IRS derivative of decreasing marginal and average costs. We focus on the survival of firms, and as such ours is decidedly *not* a model of endogenous technical change. That said, persistent market competition amidst increasing returns is relevant to the technical change literature. The solution we propose retains homogeneous goods and agents, but dispenses with such general equilibrium devices as the Walrasian Auctioneer and the Law of One Price in the market. We address two puzzles simultaneously, using price dispersion to make enduring competition possible under IRS. By allowing a distribution of prices in the market, we return to an older vision of economics in which markets are seen

[1] The most notable exception occurs if the model is initiated with a large technology gap, where growth is strictly decreasing with competition.

as embedded within a society characterized by connected agents of limited information and abilities. This version of the economic way of thinking is found in Adam Smith with non-uniform prices (Aspromougos, 2007) and was revived in our era by George Stigler in his celebrated theory of search (Stigler, 1961). The distribution of prices motivates search. This leads inevitably to the question of what keeps the distribution from collapsing to the price point mass that characterizes perfect competition, monopoly, and the Law of One Price. This possibility of multiple prices would appear to be further strained in a model where production enjoys increasing returns. Competitive markets should collapse to a single firm, as productive capacity increasing with size is unbounded; the leading firm inevitably dominates the entire market.

A distribution of prices can only survive so long as a distribution of firms can coexist to provide for it. This multiplicity of firms in our model is fostered by a never-ending innovative race with firms constantly vying for position. At any moment there is a single leader, but this dominance is precarious and subject to turnover. Competition remains intact so long as firms can survive their time at the bottom and are limited in their capacity to bankrupt other firms when they take their turn at the top. It is the confluence of limited agent information, costly search, a distribution of prices, ability to survive when business is slow, and turnover at the technological top that allows the competitive market to remain robust in the face of enduring and ubiquitous increasing returns to scale.

We propose that the microfoundations of models with IRS can be better reconciled with a competitive marketplace without the *ex ante* heterogeneity assumptions of monopolistic competition models.[2] Where many have employed the Dixit–Stiglitz (1977) model of monopolistic competition derivative of relaxing the assumption of good homogeneity, we instead construct a model of a restless market, which we will call Stigler competition, retaining the homogeneity of goods and instead relaxing the assumption of unlimited (sufficient) information.[3] Increasing returns is a long run concept. Multiple firms are able to survive from one short-run to the next because of the limitation of knowledge excludability over time.[4]

2. Monopolistic competition and imperfect information

When Young (1928) attempted to reinterest economists in the productive potential of Adam Smith's division of labor, he paid tribute to Marshall's distinction between internal and external economies of scale as offering a "safeguard against the common error of assuming that whenever increasing returns operate there is necessarily an effective tendency towards monopoly." From Smith to Young the goal has been to explain economic progress under competitive conditions. Romer (1994) offers a retrospective on endogenous growth theory in which he succinctly outlines the challenges faced by growth theorists, noting that it was "...obvious that growth theorists would have to... abandon the assumption of price-taking competition." The approach taken by Romer (1990) was to employ the Dixit–Stiglitz model of monopolistic competition, which is sufficient to allow for persistent market decentralization under IRS. While there exist reasonable arguments and evidence for the existence of market power held by individual firms, the rapid adoption of monopolistic competition models in growth theory appears, at least in part, motivated by its offer of tractability under IRS. Stigler (1949) concurred with Chamberlin's criticisms of perfect competition, but suggested that a model would be better served by dropping the stationarity assumption and retaining the kind of competition that is inherent to a market for a homogeneous good. We contend that many of the assumptions underpinning monopolistic competition are not necessary to retain decentralized, competitive markets under IRS.

Price dispersion is an empirical reality that appears unwilling to go away, despite seemingly enormous reductions in the cost of information. Even when prices follow the expected pattern of dispersion diminishing with decreasing costs of information, such as in online book sales, dispersion remains statistically different from zero (Clay et al., 2001). Further, there is evidence that over a sufficient period of time, price differences are endemic to a sector, without being systematically correlated to the firms occupying that sector or subtle differences in product (Baye and Morgan, 2004; Baye et al., 2005; Chen, 2006). This persistence of price dispersion calls into question how to test theories that require that identical goods satisfy the Law of One Price.

It is amusing that price dispersion, and all the theoretical consternation it can cause, is rather intuitive, almost *obvious*, in light of Stigler's terribly uncontroversial theory of search. The simple logic of an activity whose continued participation is governed by its marginal costs and benefits is nearly beyond reproach. Extensions and application of this theory have been numerous (for specific theories and reviews of the literature, see Acemoglu and Shimer, 2000; Manning and Morgan, 1982; Reinganum, 1982; Rogerson et al., 2005). Search theory and inflation have overlapped in efforts to formalize our understanding of price dispersion and its effect on efficiency (Benabou, 1988, 1992; Diamond, 1971). Benabou identifies instances where price dispersion is potentially welfare enhancing, through a reduction in the number of firms and rent dissipation. While Benabou's model differs from ours, our model similarly attributes welfare enhancing properties to price dispersion.

---

[2] *Ex ante* heterogeneity, that of exogenously imposed attributes, is to be contrasted with *ex post* heterogeneity that emerges endogenously in model from agent choices and random outcomes.

[3] Stigler offers a promissory note on just such a model of competition in a note to his 1949 lecture on monopolistic competition (Stigler, 1949, p. 19 note 3, quoted at the beginning of this paper).

[4] This restriction can be construed as a legal or moral framework that affects intertemporal reciprocity akin to the property rights which allow self-interested agents to trade (Levy, 1992; Peart, 2006).

Addressing the puzzle of multiple prices in a competitive market, Reinganum (1979) demonstrated that imperfect information and search can foster equilibrium price dispersion in a market model. Similarly, there is a family of models built with imperfect information that generate monopolistically competitive outcomes (Braverman, 1980; Salop, 1976; Salop and Stiglitz, 1977, 1982; Von zur Muehlen, 1979). These models generate multiple price equilibria, where each customer in the market can in principle be faced with a price unique to her. While they do successfully move beyond the assumption of commodity heterogeneity to retain prices in the market above the perfectly competitive price, they collapse to a single price under conditions of internal IRS. The collapsing of multiple (two) price equilibria models of monopolistic competition, such as Salop and Stiglitz (1977), occurs because the portion of the consumer population that will purchase at the non-competitive price is a function of the steepness of the early portion of the U-shaped cost curve. If the cost curve is monotonically declining, however, there are not any customers willing to pay a higher price, and the "competitive" price will only be offered by a single, monopolistic firm.[5] The models generate two-price equilibria that are a result of having two types of consumers and a binary information outcome (complete information or zero information); when information can be acquired piecemeal there are as many prices in the market as there are types of consumers (Butters, 1977; Salop, 1976; Von zur Muehlen, 1979). The reliance of these models on consumer heterogeneity, specifically referenced to as heterogeneity of *rationality* by Salop and Stiglitz (1977) is questionable, potentially distressing.[6]

Without a model of a market with homogenous goods and customers we are often at a loss for understanding the competitive outcome and its ramifications for market efficiency. Perfect product differentiation across firms builds in a decentralized market outcome *ex ante*. Product homogeneity, on the other hand, leaves open the possibility for a single firm outcome. As such, it is only under conditions of product homogeneity that we can arrive at theoretical conclusions regarding the effects of a construct, such as IRS, on the competitive outcome of a market. Further, given that sectors, even those heavily commoditized, are typically occupied by multiple firms, the conclusion that a model with increasing returns is compatible with what we observe in the real world requires it to generate decentralized markets under conditions of product homogeneity.

3. A model of Stigler competition

The story told in our model is of firms constantly vying for technological leadership and the prospective rents it entails, while at the same time struggling to keep ahead of the encroaching shadow of bankruptcy and failure. Innovation is rewarded with temporary rents and increased prospects for customers, but the expiration of private knowledge excludability challenges each firm with goalposts forever being moved farther away. It is the costliness of information acquisition facing customers, and their subsequent heterogeneity of information, that allows technologically lagging firms to survive today, and contribute to a competitive market tomorrow.

Economic time as formulated in Viner (1931) is dichotomized between a short run in which there are fixed inputs and a long run in which all can vary. The short run in economic time is instantiated in our model as a single time "step," indexed t, where $t \in [1 \ldots T]$. During a step each firm sets price equal to the previous period's marginal cost and draws a new private technology flow from a distribution.[7] The difference between the last period's marginal cost and price provides a quasi-rent for the firm. There is a cooperative element in our model that has no counterpart in an industry in which only low cost firms survive. Successful innovation is excludable, and in turn rewarded, for a period before the innovation is shared. Firms post their prices, creating a publicly available information set over which customers individually engage in a costly search, eventually choosing the firm with the lowest known price, and expending their remaining endowment. Firms that fail to attract customers declare bankruptcy and exit the model. Innovation, here exogenous to the model, is motivated both by the need to attract customers with a sufficiently low price and the prospect of rents derivative of temporarily excludable knowledge. While the short run exists in any single time step t, the long run is emergent across multiple time steps. Given this time structure, the model is dynamic in nature, but is strictly absent any traditional equilibrium. It is, rather, a disequilibrium model, where periods of stationarity and broken ergodicity are possible, even common, but no permanent steady state can be achieved other than the potential collapse to a single monopolistic firm.

The model is composed of two vectors of agents, customers

$$\begin{aligned} &C = [1, 2, \ldots, n] \\ &i \in C \end{aligned} \tag{1}$$

[5] Braverman (1980) brings a measure of cohesiveness to this family of models, constructing a model that retains *equally* rational agents facing *different* search costs that generates price dispersion, but the model still converges to a single price under increasing returns to scale.

[6] Our model constructed here requires costly, and in turn imperfect, information. Imperfect information is a form of bounded rationality (Simon, 1955), but this limitation is homogenous across agents, and so is consistent with analytic egalitarianism. No agent, economist or otherwise, is better than any other. A definition of analytical egalitarianism is found in Peart and Levy (2005). Harold Demsetz, frequently a critic of monopolistic competition, was especially keen on claiming that the differences between competition and monopolistic competition were trivial, and that many of the efficiency claims made by advocates of Chamberlin's monopolistic competition model were dependent on assumptions of free information (Demsetz, 1968).

[7] Our focus on lagged marginal cost and the attendant quasi-rent for innovation allows us to address a wider range of production technology than simply constant returns over L and K and so take advantage of the possibility of cooperation with the development of technology amongst the firms in a strictly competitive industry. The possibility of cooperation is precluded if only the lowest cost firm survives at any moment.

and firms

$$\begin{aligned} &F = [1, 2, \ldots, m] \\ &j \in F \end{aligned} \tag{2}$$

where each customer, i, purchases q_i^j units from the firm, j^*, offering the lowest price known to her during time step t. All variables that are not exogenously set vary across time steps. For ease of explication, we will not include t as a subscript except when previous time steps ($t-1$) are relevant. Agent search occurs within each time step t, in sub-steps $\tau = 1 \ldots m$ where each increment of τ represents an act of search by the agent.[8] Agents search over the price set Ω, where $\Omega_{i,\tau}$ is the subset of prices known to customer i after τ search efforts. How many units a customer buys is a function the uniform wage rate, w, the subunits of time (within each time step t) she dedicates to wage yielding labor, h_i^L, and the lowest price known to her, $p_i^{j^*}$, where

$$\begin{aligned} &\Omega \equiv \left\{p^1 \ldots p^m\right\} \\ &\Omega_{i,\tau} \subset \Omega \\ &p_i^{j^*} = \min\{p_i^j | p_i^j \in \Omega_{i,\tau}\} \quad \forall i \end{aligned} \tag{3}$$

$$q_i^j = \frac{h_i^L \cdot w}{p_i^{j^*}} \quad \forall i \tag{4}$$

and customers have a uniform time endowment of H within each step t.

$$h_i^L + h_i^S = H \quad \forall i \tag{5}$$

with time allotted to labor, h_i^L, and price searching, h_i^S. Customers have simple, linear preferences, and therefore do not smooth income across time steps or form expectations for the future.

Each firm, j, can produce Q_j, where

$$Q_j = (G + R_j)K_j^{\alpha} L_j^{\beta} \quad \forall j \tag{6}$$

with inputs of public knowledge, G; private knowledge, R; labor, L[9]; and capital, K. We will consider two interesting cases. First, $\alpha + \beta = 1$, where firms exhibit constant marginal costs with regards to costly inputs L and K. Changes in available knowledge inputs (G and R) shift the CRS production function over L and K (Terleckyj, 1974). The movement from SRAC(R_t) to SRAC(R_{t+1}) (Fig. 1) is technological innovation (Levy and Terleckyj, 1989), emerging IRS in the long run (see Fig. 1). Second, for a robustness check we consider $\alpha + \beta > 1$. Here firms exhibit decreasing marginal costs with regards to costly inputs L and K, exhibiting IRS in both the short and long run. Q_j is produced subject to the resource constraint:

$$TC_j = wL_j + rK_j \quad \forall j \tag{7}$$

It is assumed that all firms have access to sufficiently liquid capital and labor markets such that they each employ the cost minimizing combination of inputs to produce as much as the market demands. Firm production is standard cost-minimization constrained optimization, with the Lagrangian

$$\mathcal{L}_j = rK_j + wL_j + \lambda(Q_j - A_j {K_j}^{\alpha} {L_j}^{\beta}) \quad \forall j \tag{8}$$

where Q_j is the market order the firm must fill. For simplicity of explication:

$$\begin{aligned} &A_j \equiv G + R_j \\ &Z \equiv \frac{r\beta}{w\alpha} \end{aligned} \tag{9}$$

[8] There are m firms, and thus m prices over which to potentially search. If the cost of a unit of search, Δh, equaled zero, all agents would continue search until τ equaled m.

[9] To preempt any confusion, we note that that labor supply is not a function of the number of customers (n) in the market, nor the labor hours (h_l) customers have left over within a time step after their price search has been executed. The model assumes a large, exogenous, labor pool from which firms can draw, that is available at a uniform wage rate w. The short run, intra time step decisions of firms are, as we have noted in the text, part of a partial, rather than general, equilibrium.

and, as such, the f.o.c. are

$$\begin{aligned} \frac{d\mathcal{L}_j}{dK_j} &= r - \lambda A_j \alpha K_j^{\alpha-1} L_j^{\beta} = 0 \\ \frac{d\mathcal{L}_j}{dL_j} &= w - \lambda A_j \beta K_j^{\alpha} L_j^{\beta-1} = 0 \\ \frac{d\mathcal{L}_j}{d\lambda_j} &= Q_j - A_j K_j^{\alpha} L_j^{\beta} = 0 \end{aligned} \tag{10}$$

Efficient production employs production in the ratio

$$K_j = \frac{L_j}{Z} \tag{11}$$

where

$$L_j = \left(\frac{Q_j}{A_j}\right)^{1/(\alpha+\beta)} Z^{\alpha/(\alpha+\beta)} \tag{12}$$

For simplicity, we assume that the uniform wage rate, w, and cost of capital, r, are exogenous to the model.[10]

Knowledge is produced exogenously. Firms produce using two knowledge input factors, one public, G; the other private, R. In our model private knowledge becomes public in one period, so the stock R is in this case a degenerate one period stock, i.e., a flow. Public knowledge, G, is the sum of the superior stock of private knowledge from each proceeding term (Eq. (13)).[11] Each firm's private stock is eliminated (made public domain) and must be recreated through research and development (R&D). In this manner firms are participating in an innovative race each turn, and after that turn is complete and quasi-rents from excludable knowledge reaped, they proceed to share their knowledge, simplified such that the "best" innovation (highest value of R) is made public and added to the public knowledge stock. R&D is a random normal process whose mean is a fraction, f, of the public stock of knowledge (Eq. (14)). Negative values represent failure in the R&D process. As such, innovation is random across firms (no firm is any smarter, or investing more, than any other). The sole input is the existing body of public knowledge, G, such that the scale of R relative to G is consistent over time. Values of R are truncated at zero, to avoid having R&D failures impact production

$$G_t = G_{t-1} + \max\{R_{1,t-1} \ldots R_{m,t-1}\} \tag{13}$$

$$R_{j,t} = N\left(Gf, \frac{Gf}{4}\right) \quad \forall j \tag{14}$$

The price a firm offers in the market is the marginal cost of production from the previous time step, $t-1$, where

$$\begin{aligned} p_{j,t} &= MC_{j,t-1} \quad \forall j \\ MC_{j,t} &= \left(w + \frac{r}{Z}\right) \frac{1}{A_{j,t}} Z^{\alpha} \\ \pi_{j,t} &= \left(w + \frac{r}{Z}\right) Z^{\alpha} \left(\frac{1}{A_{j,t-1}} - \frac{1}{A_{j,t}}\right) Q_{j,t} \end{aligned} \tag{15}$$

The use of a one-period lagged marginal cost requires discussion. By our knowledge sharing assumption, knowledge is private for one period and then public, we are allowing only publically available knowledge to influence price. The firm's specific knowledge created in the current period influences the firm's quasi-rents. Further, given the lag in pricing, firms that fail to innovate still stand to earn quasi-rents, however smaller than those possible with private innovation. One might well question the implicit supposition that firms will co-operate in developing knowledge that will go public so quickly, especially when rents are still possible. We address in Section 5.3 reciprocity and the prospects of firms that free ride on shared knowledge.

Customers employ a simple search heuristic to find a suitably low price in the market. Prices change after each step t, requiring a unique search effort within each turn. Within a time step, t, all agents have uniform endowments of subunits of time H which they divide between search and labor (see Eq. (5)). Search occurs in a sequence of random draws, $\tau = 1 \ldots m$, from the prices subset of prices unknown to the consumer, $\Omega_{i,\tau}$. Each draw expends a specified amount of time, Δh, such that

[10] The results of the model remain robust if w and r are set to the average value of their marginal products $w = \left(\left(\sum_{j=1}^{m} dQ_j/dL_j\right)/m\right) \cdot p_i^{j*}$, and $r = \left(\left(\sum_{j=1}^{m} dQ_j/dK_j\right)/m\right) \cdot p_i^{j*}$, but this invites an additional level of complexity with regards to agent decision making and additional labor hours.

[11] Alternative model specifications, conducted but not reported here, allow for a longer period of private accumulation. Long periods of excludability led to faster rates of decline amongst the population of firms, but did not change the results in any qualitatively noticeable way.

$h_s = \tau \cdot \Delta h$. After some preliminary number of draws,[12] customers will continue to search for prices so long as the marginal benefit to expending an additional amount of time, Δh, on search exceeds the marginal costs of foregone wages ($E(\Delta q_i) > 0$ where $\Delta q_i = E(q_{i,\tau+1}) - q_{i,\tau}$). Within this calculation the agent employs a expected price heuristic where the *new* low price that can be expected from additional search, $p^{j^{**}}_{i,\tau+1}$, is the percentage difference between the current lowest (known) price, $p^{j^*}_{i,\tau}$, and the second lowest price, $p^{j'}_{i,\tau}$.[13]

$$\begin{aligned} E(p^{j^{**}}_{i,\tau+1}) &= p^{j^*}_{i,\tau} \cdot \left(1 - \frac{p^{j'}_{i,\tau} - p^{j^*}_{i,\tau}}{p^{j'}_{i,\tau}}\right) \quad \forall i \\ E(\Delta q) &= \frac{w \cdot (H - (\tau+1)\Delta h)}{E(p^{j^{**}}_{i,\tau+1})} - \frac{w \cdot [H - (\tau \cdot \Delta h)]}{p^{j^*}_{i,\tau}} \end{aligned} \tag{16}$$

Customers will execute some number of searches, τ, such that $\tau < m$. Each act of search adds to h_i^S and diminishes h_i^L by Δh. When search is concluded, each customer places an order $q_i^{j^*}$ units from the firm offering the lowest found price, j^*, for as many units as she can afford given her total income resultant of her wage w and remaining subunits of time h_i^L. After all agents have placed their orders, each firm produces

$$\begin{aligned} q_j &= \sum_{i=1}^{n} q_i^f \cdot \theta \quad \forall j \\ &\text{where } \theta \begin{cases} 1 & \text{if } f = j^* \\ 0 & \text{if } f \neq j^* \end{cases} \end{aligned} \tag{17}$$

3.1. Simulation steps

The model is organized into steps and sub-steps.[14] A run of the model is constituted by an initialization ($t = 0$) followed by a set number of model steps, during which every agent is activated in random order, as arranged by the model sub-steps. The sub-steps are ordered as follows:

(1) All private knowledge capital is made public; the largest value of R, from the set of firms, F, from the previous time step is added to the cumulative stock of public capital, G (see Eq. (13)).[15]
(2) Each firm sets its price in the market to its marginal cost in the previous time step (see Eq. (15)).
(3) Customers are activated in random order.[16] When activated, a customer executes τ searches over the set of all available prices, continuing one price at a time while the expected value of an additional unit of search is positive. Customers always start their search process by including their previously chosen producer in their initial search set, followed by a second random draw from the set of unknown prices.
(4) Firms are activated in random order. When a firm is activated, it (a) conducts research, establishing a the new value for its private stock of knowledge, R_t, (see Eq. (14)), and then (b) produces Q_j (see Eq. (17)).
(5) If a firm is unable to procure any market orders, it may go bankrupt. Bankruptcy results when a firm has B consecutive turns with zero market orders, and in turn, zero profits.[17]

The model, as constructed, is characterized by a homogenous good (Q), purchased by consumers, C, produced by firms, F. Customers and firms are homogeneous in capacity, but are heterogeneous in information/knowledge. Customers acquire heterogeneous information sets through search, while firms acquire heterogeneous private knowledge stocks through random draws from a normal distribution identical across firms. Increasing returns to scale emerge as a long run property from the technological innovation required for firm survival. Costly search by customers and the sharing of old knowledge keep firms profitable and market competition robust.

---

[12] As noted later, agents always initiate their search effort by starting with the price offered by the firm they chose in the previous time step since we suppose it is costless to observe the price at the firm with which one deals.

[13] This is admittedly a very aggressive and optimistic heuristic from the customer's point of view. Customers who aggressively search, however, make the environments less hospitable to persistent competition and so biases the model against any novel result.

[14] The model is written in Java using the MASON agent modeling library (Luke et al., 2005). The step/sub-step construct is built into the MASON model scheduling system.

[15] This step is altered when testing under conditions without public sharing of private knowledge shocks (see Eq. (18)).

[16] Random agent activation is best practice in multi-agent modeling. Sequential (non-random) agent activation has been demonstrated to generate various "artifacts," emergent properties derivative of sequence rather than model elements being tested (Axtell, 2001).

[17] This bankruptcy rule reflects both a technical and a substantive claim. Technically, our discrete model only allows integer sales. Substantially, B uniform across all firms holds in the case in which no firm free rides in the production of technology so the survival of all firms is the same B turns with zero market orders. We will drop this assumption below when we impose a cost on innovating firms by giving them a lower B than free riding firms.

Table 1
Model parameters.

Parameter	Context/related function	Value
H	Agent time endowment	1
m	Starting number of firms	500
n	Number of customers	5000
α	$Q_j = (G + R_j)K_j^{\alpha} L_j^{\beta}$	[0.5, 1]
β	$Q_j = (G + R_j)K_j^{\alpha} L_j^{\beta}$	[0.5, 1]
w, r	Wage, capital rent	1
G	Initial public stock	100
B	Buffer stock (turns to bankruptcy)	2
f[a]	$R_{j,t} = N\left(G \cdot f,\ \frac{G \cdot f}{4}\right)$	0.05

[a] The shapes of the functions in Figs. 1 and 2 are neutral to the size of the mean rate of innovation, f. The growth rates that emerge are typically in excess of 8 percent—an impressive rate of aggregate growth, but this rate is strictly connected to the rate of innovation, which is exogenously set. Future extensions of the model will likely endogenize innovation, making aggregate growth outcomes more interesting.

4. Experiment

We conducted two separate experiments, defined by the nature of returns to scale. The first allowed for IRS emergent in the long run, under conditions of constant marginal cost ($\alpha + \beta = 1$). The second featured short-run IRS, under conditions of decreasing marginal cost ($\alpha + \beta > 1$). Each experiment entailed the model being run 1000 times, with 1000 time steps constituting a run. Settings reflect an effort to construct a neutral control setting for testing the impact of search costs (Δh) and knowledge stock sharing. The number of turns without an order it took for a firm to go bankrupt, B, was tested extensively. As should be expected, the rate of firm bankruptcy declines as B increases. For the sake of both simplicity and an effort to the make the model *less* hospitable to competitive outcomes, $B = 2$ in all model runs reported, making surviving 1000 turns a sufficiently challenging task for firms (see Table 1).

For the sake of simplicity, we will refer to a market as competitive when it has more than one firm. Our model does not include any strategic behavior on the part of firms, and offers no insight into potentially oligopolistic or collusive behavior.

The structure of the demise of bankrupted firms is a variant of the gambler's ruin, where a sequence of low probability events is inevitable in the fullness of time (Garrod and Miklius, 1990; Wilcox, 1971). This outcome can be equated to a *broken ergodicity*, where all outcomes between m and 1 will eventually be sampled, but a specific subset of the values, in this case an extended market stationarity of $1 < \#\,firms < m$, can persist for long time scale with a high probability (Axtell et al., 2001; Palmer, 1989).

5. Results

5.1. $\alpha + \beta = 1$

Competitive outcomes emerge from this model when search costs exceed a necessary threshold. In Fig. 2 we can observe the direct relationship between search cost (Δh) and the number of surviving firms in the market after 1000 time steps. The

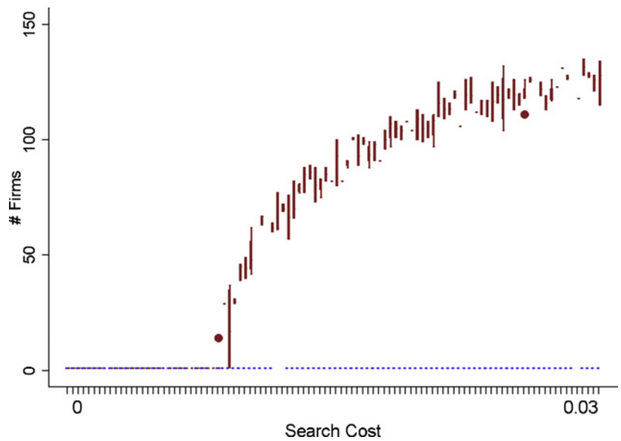

Fig. 2. Number of firms active in the market over search costs (1000 turns). Sharing "on" (upper) and Sharing "off" (lower), $\alpha + \beta = 1$, 5000 customers.

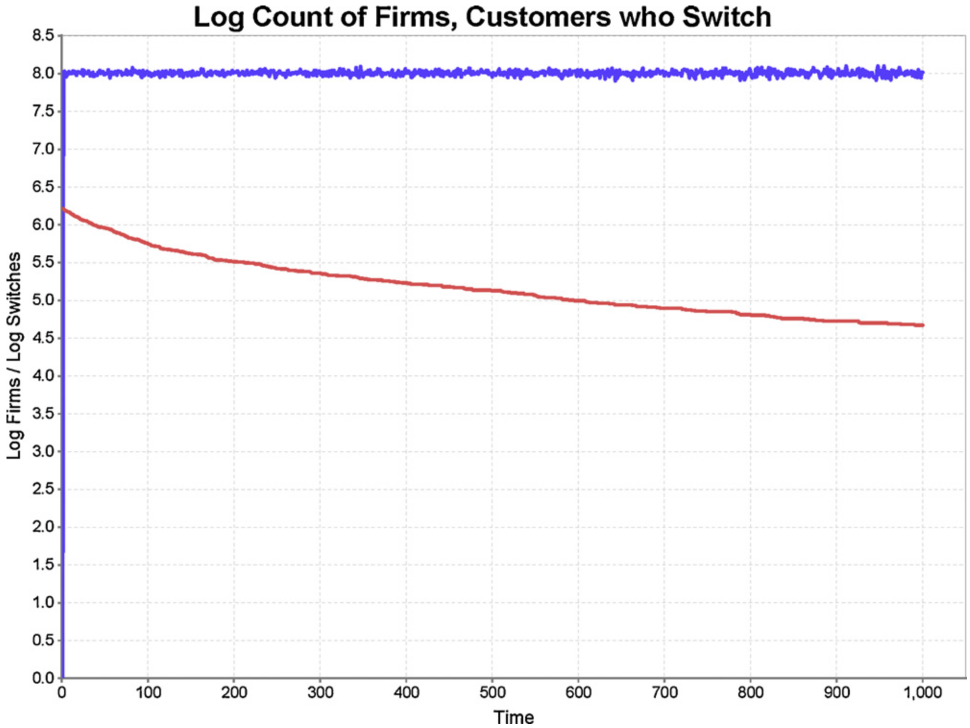

Fig. 3. Number of firms (lower), customers who switched since the previous turn (upper), $A+B=1$, search costs = 2 percent of time endowment, 5000 customers.

smallest values of Δh have scant effect; the market remains non-competitive until a threshold value is reached. Beyond this threshold, the number of firms is rapidly increasing early in search costs, with diminishing returns.

The number of firms is diminishing over time, as can be seen in the active firms chart from a single run in Fig. 3. The rate of decay, however, is diminishing as well; competitive outcomes are likely to remain for thousands of time steps.

Heterogeneity of research outcomes and the sharing of knowledge drive the results. Controlling for these functions demonstrates as much. When private research factors (R) are removed from the production function, firms become effectively homogeneous and LRAC exhibit the same constant returns to scale as SRAC; the model emerges the familiar perfect competition outcome. Knowledge sharing, conversely, is what prevents collapse to monopoly when knowledge stocks grow and LRAC exhibit increasing returns to scale. We can turn knowledge sharing "off" by constraining the public stock to zero, and replacing the innovation function (Eq. (15)) with:

$$R_{j,t} = N\left(R_{j,t-1}\cdot f, \frac{R_{j,t-1}\cdot f}{4}\right) + R_{j,t-1} \tag{18}$$

Turning sharing "off" results in a private knowledge stock that is completely excludable. Because knowledge is growing, however, LRAC are still dropping, which in a standard theoretical context leads to monopoly. The results of turning sharing "off" in our model is collapse to a non-competitive market (Fig. 2), where every individual run of the model collapsed to a single monopoly firm. This result was consistent with both emergent and short-run IRS. The importance of the possibility of knowledge sharing is not seen when customers are *assumed* to be capable of always buying from the lowest cost producer.

The extended periods of stationarity in the number of firms surviving in the market under conditions of sharing are not a product of the model achieving any sort of stasis. Customer loyalties remain dynamic throughout the model, with the percentage of the customer population that switches producers during any one turn, while fluctuating, consistently exceeding 50 percent (Fig. 3). This bustling activity actually *underpins* the stationarity of market competition. A market without fickle customers and turning tides of innovation quickly collapses to monopoly.

5.2. $\alpha+\beta>1$

Short-run IRS derivative of decreasing marginal cost present a stiffer challenge to competitive markets. Under emergent IRS, superior innovation breeds success via lower prices. When firms within the model enjoy lower costs with each unit produced, we enter a world where success breeds greater success. Innovation stands to build a juggernaut, building greater distance between the price leader and second best with each item sold.

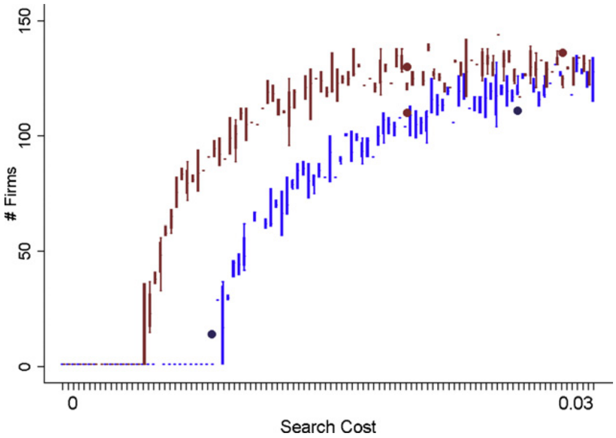

Fig. 4. Number of firms active in the market over search costs (1000 Turns). $\alpha + \beta = 1$ (upper), $\alpha + \beta > 1$ (lower).

It is not surprising, then, that the market marches toward monopoly with greater expediency with decreasing MC. What is surprising, however, is the still relatively resilient nature of the market characterized by search costs and price dispersion. Fig. 4 includes sets of box plots across search costs, where the red boxes represent runs with $\alpha + \beta = 1$ (emergent IRS) and blue boxes represent runs with $\alpha + \beta > 1$ (short-run IRS).

5.3. *Sharing, reciprocity, and free riding*

Private knowledge is excludable for one period. As we have seen without positive search costs this suffices for the industry to collapse to a single firm. Thus, with positive search costs we need to consider dimensions of competition hidden by the Law of One Price. We focus on the role of reciprocity, which is of much importance in classical economics (Peart and Levy, 2005), and is returning to the modern economics literature (Croson, 2007; Sugden, 1984). Given that the prices only reflect the public stock of knowledge, firms who innovate stand to earn quasi-rents. After a single time step, however, private knowledge is shared, and added to the public stock of knowledge. The limitations on knowledge excludability can be construed as a legal or moral framework that affects intertemporal reciprocity akin to the property rights which allow self-interested agents to trade (Levy, 1992; Peart, 2006). Firms innovate operating on the assumption that they will have the opportunity to earn rents today and share their private knowledge with other firms on the understanding that other firms will behave similarly. Innovation and sharing are made possible both by the promise of rewards to the leading firms today and the understanding that the leading firms of tomorrow will similarly share their superior knowledge tomorrow.

Earlier we took notice that quasi-rents were still possible for firms that chose not to innovate, and instead to free ride off the innovation of others, with rents solely generated by the public sharing of leading innovations from previous time steps. In order to test the possibility of firms surviving as free riders who chose not to innovate, we can augment the model to add a cost to innovation, namely a shorter number of turns until potential bankruptcy ($B_{innovators} < B_{free\ riders}$). Non-cooperation, as we model it here, allows a firm to survive longer without customers. Free-riding firms have the opportunity to reap quasi-rents, but must compete with firms who are innovating, and in turn, whose prices (lagged marginal costs) are lower. We considered the case where half the firms co-operate and engage in research and half do not. Without exception, the free riders (firms who chose not to engage in private R&D) fail to survive for long, disappearing within a handful of time steps beyond $B_{free\ riders}$. The logic is relatively simple. Those firms who free ride are, in our simple model, always the highest cost producers. Customers are likely to find lower cost producers. In fact, for non-innovators to survive, they would require cooperation among other firms to agree *not* to innovate. Such cooperation is unlikely given the additional quasi-rents to be generated from temporarily excludable new private knowledge. Innovation can be construed as an act of defection that undermines the feasibility of surviving absent innovation.

6. Discussion

There is no entry in our model, and, as such, no replacement of bankrupted firms. The robustness of competition is considerably more striking under these modeling conditions. Without replacement, there will always be firms that go out of business simply because they were unlucky one too many times in a row. As such there is, at the margin, room for entry into the market for firms whose profitability is sustainable for a significant period.

The lack of substitutes in the market for goods under monopolistic competition limits the scope of outcomes to only those with decentralized markets, rendering competitive states determined and uninformative with regard to questions of the impact of innovation on the state of competition. Our model allows for a range of emergent outcomes bounded by pure monopoly and perfect competition.[18] If we construct the model absent excludable innovation, the familiar perfectly competitive outcome emerges—the market quickly converges to a single price and firms who wish to survive must engage in price-taking behavior. If, however, a firm may innovate and produce a private technology good that is excludable, it acquires, at least, short-term market power. If the leading firm were able to retain the excludability of their private technology stock, customers would in time all discover the leading firm's lowest price and it would acquire a pure monopoly. If the excludability of the firm's technological innovation is only temporary, and potential customers operate with limited price information, then so long as firms can continue to innovate, the model has access to all points in between perfect competition and pure monopoly.

Nelson and Winter (1978), so often pioneers of evolutionary and computational simulation models, offered a model of markets and firm size with relation to productivity growth and technological imitation which generated realistic distributions of size across firms. Their experimental results demonstrate the importance of private knowledge excludability in determining market concentration. They acknowledge that these results are weakly realized, but this is likely a result of the exclusive use of (1) constant, as opposed to increasing, returns to scale, and (2) decision rules (heuristic), as opposed to optimization, on the part of firms. Their insights into Schumpeterian mechanisms in market competition and the role of private and public knowledge in market concentration are further validated by the outcomes of our model. While their findings offer insight into the distribution of firm size and why markets concentration tends to condense away from the Walrasian perfect competition outcome, our corresponding model offers insight into why they do not collapse further to single firm monopolies.

7. Conclusion

We have constructed a model in which price dispersion and temporarily excludable knowledge allow firms to survive from one short run to the next in a world of increasing returns to scale. The market remains competitive in the long run, distinguished by a restless disequilibria, where firms are always approaching, but never settling into uniform, marginal cost pricing, save for a distant, monopolistic future likely only reachable if market entry is held at bay. We conclude by emphasizing the limitations of our model. The primary form of increasing returns to scale emphasized in this paper comes from past knowledge shifting a production function which exhibits constant marginal costs in the short run where labor and capital inputs can vary. Knowledge in our account is exogenous, allowing for equivalent average and marginal cost of inputs. This device obviates any need to appeal to monopolistic competition's exclusive focus on average cost pricing. The firms in our model are not strategic. They do not invest in the creation of knowledge for any other objective than survival from one short run to the next. We have interpreted the temporary excludability property as intertemporal reciprocity, but we have offered no insight into whether this is instantiated as either positive law (patents) or in moral obligation. We would be pleased to discover that our result is more general than we are able to show. The model might be able to shed some light on the costs of high inflation by introducing a source of price dispersion that is not reflective of the underlying progression of innovation.

We are left with a puzzle. How is it that a 19th century claim – the "law of one price" – was read back into the *Wealth of Nations?* Adam Smith is rather clear that at most there is a law of one *perceived* price (1981 [1776] I.7 ¶ 11). It is perhaps not a coincidence that Stigler's search theory, which offers a nice way to bring perception and reality together, was proposed by one of the discipline's great authorities on Adam Smith.

References

Acemoglu, D., Shimer, R., 2000. Wage and technology dispersion. The Review of Economic Studies 67, 585–607.
Aghion, P., Harris, C., Howitt, P., Vickers, J., 2001. Competition, imitation and growth with step-by-step innovation. The Review of Economic Studies 68, 467–492.
Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.
Aspromougos, T., 2007. Adam smith's treatment of market prices and their relation of 'supply' and 'demand'. History of Economic Ideas 15, 27–58.
Axtell, R., 2001. Effects of interaction topology and activation regime in several multi-agent systems. In: Goos, G., Hartmanis, J., Leeuwen, J.v. (Eds.), Multi-agent-based Simulation. Springer, Berlin/Heidelberg, pp. 33–48.
Axtell, R., Epstein, J.M., Young, H.P., 2001. The emergence of classes in a multi-agent bargaining model. In: Durlauf, S.N., Young, H.P. (Eds.), Social Dynamics. The MIT Press, Cambridge, MA, pp. 191–212.
Baye, M.R., Morgan, J., 2004. Price dispersion in the lab and on the internet: theory and evidence. The RAND Journal of Economics 35, 449–466.
Baye, M.R., Morgan, J., Scholten, P., 2005. Information, search, and price dispersion. In: Hendershott, T. (Ed.), Handbook on Economics and Information Systems. Elsevier, Amsterdam, pp. 323–330.
Benabou, R., 1988. Search, price setting and inflation. The Review of Economic Studies 55, 353–376.
Benabou, R., 1992. Inflation and efficiency in search markets. The Review of Economic Studies 59, 299–329.
Blundell, R., Griffith, R., Reenen, J.V., 1995. Dynamic count data models of technological innovation. The Economic Journal 105, 333–344.
Blundell, R., Griffith, R., Van Reenen, J., 1999. Market share, market value and innovation in a panel of British manufacturing firms. Review of Economic Studies 66, 529–554.
Braverman, A., 1980. Consumer search and alternative market equilibria. The Review of Economic Studies 47, 487–502.
Butters, G.R., 1977. Equilibrium distributions of sales and advertising prices. The Review of Economic Studies 44, 465–491.

[18] Or, perhaps more accurately stated, competition asymptotically approaching perfection. While the market remains well-populated, true MC pricing remains, here, constantly being held at bay by innovative firms acquiring rents for the most recent knowledge advancements.

Chen, J., 2006. Differences in average prices on the internet: evidence from the online market for air travel. Economic Inquiry 44, 656–670.
Clay, K., Krishnan, R., Wolff, E., 2001. Prices and price dispersion on the web: evidence from the online book industry. Journal of Industrial Economics 49, 521–539.
Croson, R.T.A., 2007. Theories of commitment, altruism, and reciprocity: evidence from linear public goods games. Economic Inquiry 45, 199–216.
Demsetz, H., 1968. Do competition and monopolistic competition differ? Journal of Political Economy 76, 146–148.
Diamond, P.A., 1971. A model of price adjustment. Journal of Economic Theory 3, 156–168.
Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. The American Economic Review 67, 297–308.
Garrod, P., Miklius, W., 1990. Modeling the timing of business firm exits. Small Business Economics 2, 141–153.
Levy, D.M., 1992. The Economic Ideas of Ordinary People: From Preferences to Trade. Routledge, London, New York.
Levy, D.M., 2007. The puzzle of Jevons. History of Economic Ideas 15, 165–171.
Levy, D.M., Terleckyj, N.E., 1989. Problems identifying returns to R&D in an industry. Managerial and Decision Economics 10, 43–49.
Luke, S., Cioffi-Revilla, C., Panait, L., Sullivan, K., Balan, G., 2005. Mason: a multiagent simulation environment. Simulation 81, 517.
Manning, R., Morgan, P.B., 1982. Search and consumer theory. The Review of Economic Studies 49, 203–216.
Nelson, R.R., Winter, S.G., 1978. Forces generating and limiting concentration under schumpeterian competition. The Bell Journal of Economics 9, 524–548.
Palmer, R., 1989. Broken ergodicity. In: Stein, D. (Ed.), Lectures in the Sciences of Complexity, Santa Fe Institute Studies in the Sciences of Complexity. Addison-Wesley Longman, Reading, MA, pp. 669–735.
Peart, S., Levy, D.M., 2005. The "Vanity of the Philosopher": From Equality to Hierarchy in Postclassical Economics. University of Michigan Press, Ann Arbor.
Peart, S.J., 2006. Private email to the Co-Authors. April 25th.
Reinganum, J.F., 1979. A simple model of equilibrium price dispersion. The Journal of Political Economy 87, 851–858.
Reinganum, J.F., 1982. Strategic search theory. International Economic Review 23, 1–17.
Rogerson, R., Shimer, R., Wright, R., 2005. Search-theoretic models of the labor market: a survey. Journal of Economic Literature 43, 959–988.
Romer, P.M., 1990. Endogenous technological change. The Journal of Political Economy 98, S71–S102.
Romer, P.M., 1994. The origins of endogenous growth. The Journal of Economic Perspectives 8, 3–22.
Salop, S.C., 1976. Information and monopolistic competition. The American Economic Review 66, 240–245.
Salop, S.C., Stiglitz, J.E., 1977. Bargains and ripoffs: a model of monopolistically competitive price dispersion. The Review of Economic Studies 44, 493–510.
Salop, S.C., Stiglitz, J.E., 1982. The theory of sales: a simple model of equilibrium price dispersion with identical agents. The American Economic Review 72, 1121–1130.
Simon, H.A., 1955. A behavioral model of rational choice. The Quarterly Journal of Economics 69, 99–118.
Smith, A., 1981 [1776], The Wealth of Nations Indianapolis: Liberty Fund.
Stigler, G.J., 1949. Five Lectures on Economic Problems. London School of Economics and Political Science; Longmans Green, London, New York.
Stigler, G.J., 1961. The economics of information. The Journal of Political Economy 69, 213–225.
Sugden, R., 1984. Reciprocity: the supply of public goods through voluntary contributions. Economic Journal 94, 772–787.
Terleckyj, N.E., 1974. Effects of R&D on the productivity growth of industries: an exploratory study. National Planning Association.
Viner, J., 1931. Cost curves and supply curves. Zeitschrift für Nationalökonomie 3, 23–46.
Von zur Muehlen, P., 1979. Monopolistic Competition and Sequential Search. Division of Research and Statistics, Federal Reserve Board.
Warsh, D., 2006. Knowledge and the Wealth of Nations: A Story of Economic Discovery. W.W. Norton, New York.
Wilcox, J.W., 1971. A gambler's ruin prediction of business failure using accounting data. Sloan Management Review 12, 1.
Young, A., 1928. Increasing returns and economic progress. Economic Journal 38, 527–542.