# First International Workshop on Algebraic Geometry and Approximation Theory

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## What Are Multivariate Splines?

Peter Alfeld, University of Utah

**Abstract.** For the purposes of this talk, splines are smooth piecewise polynomial functions defined on a partition of a two or three dimensional domain into triangles or tetrahedra. I will define these objects more precisely, describe some basic question and techniques, list some known results, describe some open problems, and demonstrate my spline analysis software.

## Issues in Multivariate Polynomial Interpolation

Carl de Boor, University of Wisconsin-Madison

**Abstract.** While univariate polynomial interpolation has been a basic tool of scientific computing for hundreds of years, multivariate polynomial interpolation is much less understood. Already the question from which polynomial space to choose an interpolant to given data has no obvious answer. The talk presents, in some detail, one answer to this basic question, namely the "least interpolant" of Amos Ron and the speaker which, among other nice properties, is degree-reducing, then seeks some remedy for the resulting discontinuity of the interpolant as a function of the interpolation sites, then addresses the problem of a suitable representation of the interpolation error and the nature of possible limits of interpolants as some of the interpolation sites coalesce. The last part of the talk (if I ever get to it) is devoted to a more traditional setting, the complementary problem of finding correct interpolation sites for a given polynomial space, chiefly the space of polynomials of degree $\leq k$ for some $k$, to point out some interesting problems and conjectures there.

# What is Computational Algebraic Geometry?

Luis Garcia, Sam Houston State University

**Abstract.** In this talk, I will give an introduction to computational algebraic geometry with a view towards applications in geometric modeling. I will start by discussing how to solve a system of polynomial equations and why? I will discuss several methods to solve such systems (resultants, Groebner bases, and moving lines). Then I will discuss the topics of implicitization, elimination, and projection and their relevance in geometric modeling. In the last part of the talk, I will concentrate on toric varieties and their applications.

# Linear Precision for Toric Patches

Luis Garcia, Sam Houston State University

**Abstract.** In 2002, Krasauskas generalized the standard Bezier and tensor product patches of geometric modeling to multi-sided toric patches. These patches are based on the geometry of toric varieties and de- pend on a polytope and some weights. While these off er the promise of greater design flexibility, it is not clear whether they possess the desirable properties of the standard patches. One such property is linear precision, which is the ability to replicate a linear function. I will discuss work with Frank Sottile on linear precision. We show that every patch has a reparametrization having linear precision. The reparametrization is not rational unless the patch has a very singular geometry. For toric patches, the existence of such rational reparametrizations has an appealing mathematical reformulation in terms of Cremona transformations. Moreover, this reparametrization can be numerically computed using a standard method in statistical inference known as iterative proportional scaling.

# Manifold Splines and Discrete Ricci Flow

Xianfeng David Gu, Stony Brook University

**Abstract.** Constructing splines on general manifolds is one of the fundamental problems in geometric modeling. In the talk, we will discuss the following problems: can conventional splines be generalized on general manifolds? If impossible, what are the intrinsic obstructions? What is the lower bound of the number of extraordinary points? How can we construct a spline on a manifold with the minimal number of extraordinary point? Conventional spline schemes based on polar forms are constructed using affine invariants. Therefore, spline surfaces are invariant under parametric affine transformations. An affine structure on a manifold is an atlas, such that all the coordinate transition functions are affine. We proved that if a manifold admits an affine structure, then polar form splines can be defined on it directly. According to the characteristic class theory in topology, affine structure doesn't exists for general closed surfaces, unless the genus equals to one. Genus one surfaces and open surfaces admit affine structure. Furthermore, by removing one point, all surfaces admit affine structure. Therefore, the lower bound of extraordinary point is one. Ricci flow is the tool used for the proof of Poincare conjecture. We generalized it to the discrete setting and used it to construct affine structures for surfaces with arbitrary topologies with single extraordinary point. The construction can be generalized to find affine structures for 3-manifolds and build volumetric splines.

# Toric Surfaces in Geometric Modeling

Rimvydas Krasauskas, Vilnius University

**Abstract.** A chronological introduction to toric ideas in geometric modeling will be presented: the generalized stereographic projection of a sphere and other quadrics; real toric surfaces: singular cases, and cases with non-standard real structures; universal rational parametrizations of almost toric surfaces and their applications; toric surface patches; generalizations of Bezier surfaces; a few applications; subdivision of a toric patch into Bezier sub-patches, gluing functions; speculations on possible toric splines defined on closed surfaces of any genus.

# New Formulas for Divided Differences and Partitions of a Convex Polygon

Tom Lyche, University of Oslo

**Abstract.** The Leibniz rule for differentiating products of functions was generalized to divided differences by Popoviciu and Steffensen 70 years ago. To our surprise it was discovered that there were no analogs of a 150 year old formula for differentiating composite functions (Faa di Bruno's formula) and for differentiating the inverse of a function. In this talk I will discuss chain rules and inverse rules for divided differences. The inverse rule turns out to have a surprising and beautiful structure: it is a sum over partitions of a convex polygon into smaller polygons using only nonintersecting diagonals. This provides a new way of enumerating all partitions of a convex polygon with a specified number of triangles, quadrilaterals, and so on. The talk is based on joint work with Michael Floater.

# Some Solved and Unsolved Problems in Subdivision, Surface Parametrization and Algebraic Constraint Elimination

Jorg Peters, University of Florida Gainesville

**Abstract.** The talk outlines some recent results and new challenges concerning subdivision surfaces of continuity higher than $C^1$, approximation of subdivision surfaces, alternative subdivision strategies at high valences, nonlinear subdivision; bicubic surface constructions (sprocket and polar) the bi-split bicubic challenge; graph-based constraint elimination, and critical points in underconstrained problems.

# Capturing the Geometry in Formulae for Dimension of Spline Spaces

Larry Schumaker, Vanderbilt University

**Abstract.** One of the main reasons why the problem of finding explicit formulae for the dimension of spline spaces is so difficult is the fact that, except in special cases, the dimension depends not only on the combinatorics of the partition, but also on the exact geometry. This phenomenon is explained in detail for bivariate splines. Extensions to spherical splines and trivariate splines will also be discussed, along with open questions

# Quasi-interpolation by Polynomial Splines

Tatyana Sorokina, Towson University

**Abstract.** I will demonstrate very simple univariate, bivariate and trivariate quasi-interpolation schemes. Despite the fact that I constructed some of them, I do not know the true reason why they work.

# What Goes ~~wrong~~ Right with Interpolation in Several Variables

Boris Shekhtman, University of South Florida

**Abstract.** I will discuss the relationship between three (time permitted) problems in multivariate interpolation and algebraic geometry. The attempt will be made to use elementary example to explain everything, from starting observations to the latest (a month old) results and unresolved issues. The problems in question are: 1) What are Hermit projectors in several variables and reducible Hilbert Schemes. 2) Newton interpolation in several variables and nested ideals 3) Error formulas for ideal interpolation and unshortenable ideal basis. Questions and discussions during the talk are encouraged.